Geodesy 1 (GED203)

Lecture No: 5

GEODETIC POSITION COMPUTATION

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Review of Previous Lecture

REVIEW OF PLANE TRIANGLE FORMULAS

PLANE TRIANGLES VERSUS SPHERICAL/GEODETIC TRIANGLES

SPHERICAL TRIANGLE

SOLUTION OF SPHERICAL TRIANGLE

NAPIER’S RULE FOR RIGHT-ANGLE SPHERICAL TRIANGLES

SPHERICAL EXCESS

CALCULATION OF SPHERICAL EXCESS

ELLIPSOIDAL EXCESS

SUMMARY

What have we learnt so far?

please keep up

Overview of today's lecture

WHAT IS MEANT BY GEODETIC COMPUTATION?

TYPES OF GEODETIC PROBLEMS.

METHODS OF COMPUTATION

ACCURACY OF THE EXISTING METHODS

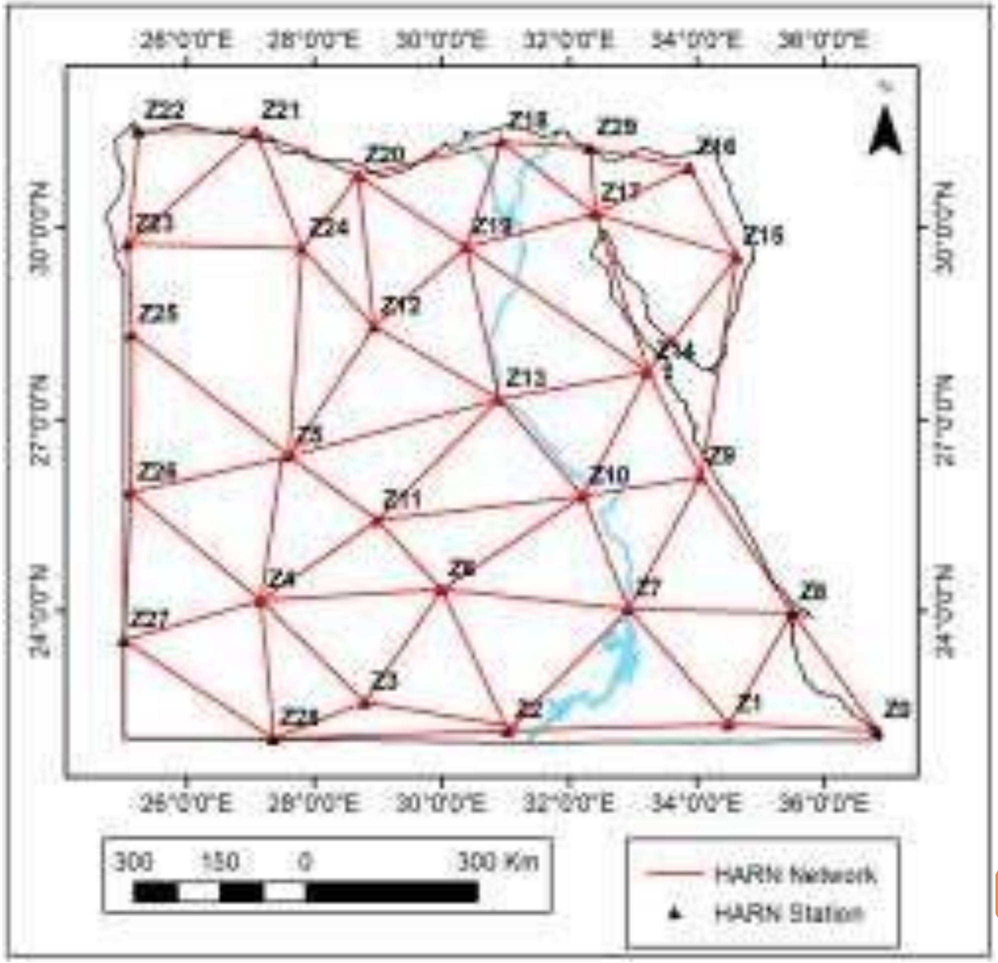
SUMMARY

Expected Learning Outcomes

- Gain a clear understanding of what is meant by the geodetic problem.
- Explore the various types of geodetic problems encountered in geodesy.
- Acquire knowledge about the different computation methods used to solve geodetic problems.
- Learn about the accuracy of existing computation methods in geodesy.
- Apply the knowledge of geodetic problems and computation methods to real-world scenarios.
- Develop critical thinking skills to analyze and solve geodetic problems effectively.
- Collaborate with peers to discuss and solve geodetic problems, share insights, and explore new approaches.

What is Geodetic Problem?

- The main problem of geodesy is the computation of the geodetic coordinates (φ, λ, h) of different points on the reference ellipsoid.
- This computation necessitates measurements of angles (H & V), azimuths, and distances.
- Using such measurements at specific locations, the coordinates of the remaining points in the network can be computed based on triangulation.



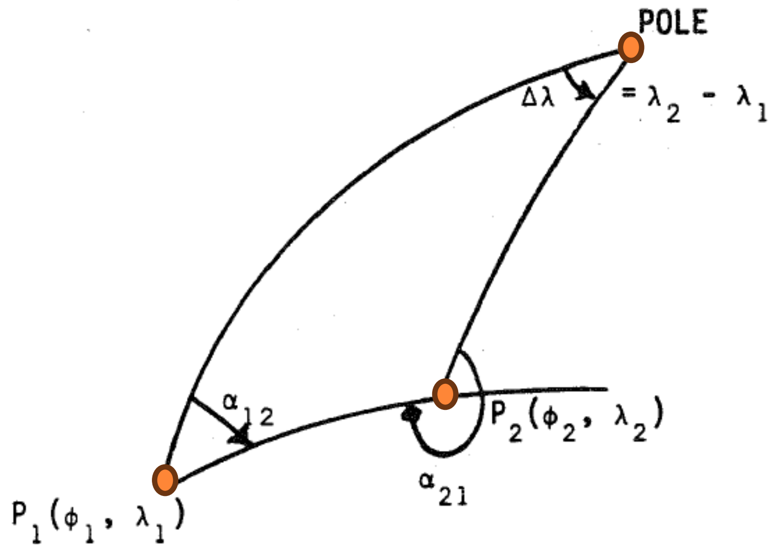
Types of Geodetic Problems

- Two cases of calculations exist:

a) ***Direct Geodetic Problem***: if the coordinates of a starting point, a distance and the forward azimuth to a second point are given and it is required to find the coordinates of the second point as well as the backward azimuth.

b) ***Inverse Geodetic Problem***: if the coordinates of the two points are known and it required to find the length of the line and its forward and back azimuth.

- The solution of either of these problems is a solution of the ellipsoidal polar triangle.



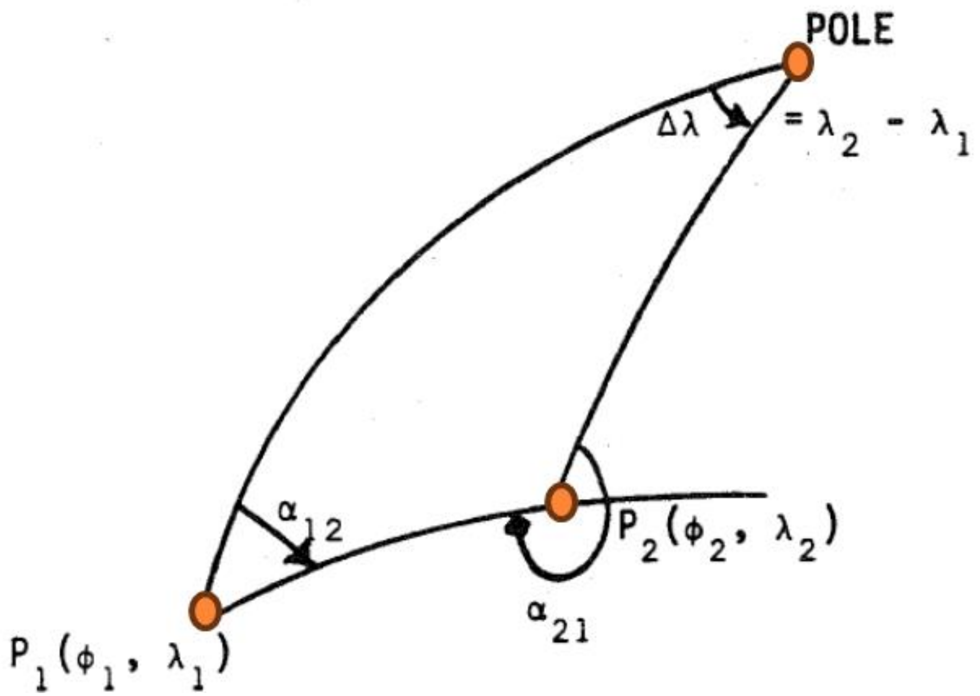
Types of Geodetic Problems

- ***Direct Problem:***

$$\varphi_2 = f_1(\varphi_1, \lambda_1, \alpha_{12}, S)$$
$$\lambda_2 = f_2(\varphi_1, \lambda_1, \alpha_{12}, S)$$
$$\alpha_{21} = f_3(\varphi_1, \lambda_1, \alpha_{12}, S)$$

- ***Inverse Problem:***

$$S = f_4(\varphi_1, \lambda_1, \varphi_2, \lambda_2)$$
$$\alpha_{12} = f_5(\varphi_1, \lambda_1, \varphi_2, \lambda_2)$$
$$\alpha_{21} = f_6(\varphi_1, \lambda_1, \varphi_2, \lambda_2)$$



How to Solve A Geodetic Problem

- On a sphere the solutions to both problems are (simple) exercises in spherical trigonometry. On an ellipsoid the computation is much more involved.
- Work on ellipsoidal solutions was carried out by for example Legendre, Bessel, Gauss, Laplace, Helmert and many others after them.
- There are many solutions for these problems.
- The solutions are classified w.r.t the distance connecting the two points and the type of this distance (*Normal Section or Geodesic*).

Methods of Computations

- **The most common methods in literature are:**

I. Clarke's formula for long lines

II. Clarke's formula for medium and short lines

III. Puissant formula and modifications of them

IV. Lilly's formula for long lines

V. Mid-latitude formula

VI. Bowring formula for geodesic lines up to 150 km

VII. The chord method

VIII. Bessel's formula

IX. The Chord Method

- These formulas are deduced use on a computational reference ellipsoid for first and second-order accuracies.
- For the case of adopting a sphere as a reference surface for geodetic computations, the simplification of these formula will yield results that are rigorous enough for third-order accuracies.

Methods of Computations - Mid-latitude Formula

- **Mid-latitude formula**

It is first published in English in 1861. These formulas rely on a spherical approximation of the earth and should only be used for points separated by less than 40 km at latitudes less than 80º. The formula is simple to use and rigorous enough for second order accuracies.

The direct problem solution is an iterative solution for d_φ.

The inverse problem is computed without iteration since φ_m is immediately available.

The accuracy of Gauss mid-latitude formulas is about 1 ppm for lines up to 100 km in length.

Methods of Computations - Mid-latitude Formula

- **Mid-latitude formula *(Direct Problem)***

Assume

$\varphi_m = \varphi_1,\ \Delta\alpha = 0$

$$\Delta\lambda = \frac{S \sin(\alpha_{12} + \Delta\alpha/2)}{N_m \cos \varphi_m)} \quad \text{.......................} (1)$$

$$\Delta\varphi = \frac{S \cos(\alpha_{12} + \frac{\Delta\alpha}{2})}{M_m \cos\frac{\Delta\lambda}{2})} \quad \text{.......................} (2)$$

$$\Delta\alpha = \Delta\lambda \times \sin\varphi_m \times \sec\left(\frac{\Delta\varphi}{2}\right) + \left(\frac{1}{12}\right) \sin\varphi_m\,(\cos^2\varphi_m)\,.\,\Delta\lambda^3 \quad \text{..........} (3)$$

METHODS OF COMPUTATIONS - MID-LATITUDE FORMULA

- **Mid-latitude formula *(Direct Problem)***

N_m ....The mean radius of curvature in the prime vertical direction

M_m ....The mean radius of curvature in the meridian direction

Then,

$\varphi_2 = \varphi_1 + \Delta\varphi$

$\lambda_2 = \lambda_1 + \Delta\lambda$

$\alpha_{21} = \alpha_{12} + \Delta\alpha \pm 180^o$

repeat the solution using $\varphi_m = \frac{(\varphi_1 + \varphi_m)}{2}$ and the new value of Δα until the value of φ remains constant.

Methods of Computations - Mid-latitude Formula

- **Mid-latitude formula *(Inverse Problem)***

$$\varphi_m = \frac{1}{2}(\varphi_1 + \varphi_2) \quad \text{.......} (4)$$

$$\Delta\alpha = \Delta\lambda \times \sin\varphi_m \times \sec\left(\frac{\Delta\varphi}{2}\right) + \left(\frac{1}{12}\right)\sin\varphi_m\,(\cos^2\varphi_m)\,.\,\Delta\lambda^3 \quad \text{.......} (5)$$

$$M_1 = S_1\cos(\alpha_{12} + \frac{\Delta\alpha}{2}) = M_m.\Delta\varphi'.\cos(\frac{\Delta\lambda}{2}) \quad \text{.......} (6)$$

$$N_1 = S_1\sin(\alpha_{12} + \frac{\Delta\alpha}{2}) = N_m.\Delta\lambda'.\cos(\varphi_m) \quad \text{.......} (7)$$

$$S_1 = \sqrt{{N_1}^2 + {M_1}^2} \quad \text{.......} (8)$$

$$\Delta\varphi' = \Delta\varphi\,[\sin\left(\frac{\Delta\varphi}{2}\right)/\left(\frac{\Delta\varphi}{2}\right)] \quad \text{.......} (9)$$

$$\Delta\lambda' = \Delta\lambda\,[\sin\left(\frac{\Delta\lambda}{2}\right)/\left(\frac{\Delta\lambda}{2}\right)] \quad \text{.......} (10)$$

Methods of Computations - Mid-latitude Formula

- **Mid-latitude formula *(Inverse Problem)***

Now compute the unknowns S, α_{12} **and** α_{21},

$$S = S_1 \left[\frac{\left(\frac{S_1}{2N_m}\right)}{\sin\left(\frac{S_1}{2N_m}\right)}\right] \qquad (11)$$

$$\alpha_{12} = \tan^{-1}\left(\frac{N_1}{M_1}\right) - \left(\frac{\Delta\alpha}{2}\right) \qquad (12)$$

$$\alpha_{21} = \alpha_{12} + \Delta\alpha \pm 180 \qquad (13)$$

METHODS OF COMPUTATIONS - PUISSANT FORMULAS

- **Puissant Formulas**

- The equations were derived by Puissant in the 18th century.
- It is a short line formula.
- Convenient to use for short lines (up to 100 km)
- Results are in third-order accuracies
- Correct to *1 ppm* for lines up to *50 km* and *40 ppm* for 250 km
- More suitable for direct problem

Methods of Computations - Bessel's Formulas

- **Bessel's Formulas**

- The derivation is based on the geodesic on the ellipsoid.
- The accuracy of Bessel formula is not limited by the separation between the two points nor by the location of the points on the earth.
- The derivation is based on parameters on a sphere and the parameters on ellipsoid.

Methods of Computations - Bowring Formulas

- **Bowring Formulas**

- Published in 1981by Bowring for geodesic lines up to 150 km in length.
- The method uses a conformal projection of the ellipsoid on a sphere called the Gaussian projection of the second kind.
- The scale factor is taken one at the starting point of the line.
- The geodesic from the ellipsoid in then projected to the corresponding line on the sphere where spherical trigonometry could be applied.

Accuracy of Direct Problem Solution Using Different Methods

	0."00001	0."0001	0."001	0."01
Legendre Series (4 terms)	30	40	80	100
Legendre Series (5 terms)	60	90	100	200
Puissant (short, 6.51)	10	10	10	10
Puissant (long, 6.40)	10	20	40	80
Bowring Chord	70	100	300	700

Maximum Error in The Solution of The Inverse Problem for Various Length Lines

Line Length	Gauss Mid Latitude		Bowring	
km	Azimuth(")	Distance(mm)	Azimuth(")	Distance(mm)
50	0".0048	4	0".0003	0.1
100	0".020	33	0".0024	1.1
200	0".083	136	0".0049	9.7

Important Resources

- https://pypi.org/project/geopy/
- https://geographiclib.sourceforge.io/
- https://pyproj4.github.io/pyproj/stable/
- https://pypi.org/project/PyGeodesy/

LET'S SUMMARIZE

THANK YOU

End of Presentation